HALBACH MAGNETS FOR CBETA AND eRHIC∗
H. Witte†, J.S. Berg, B. Parker
Brookhaven National Laboratory, Upton, NY, USA

Abstract

At Brookhaven National Laboratory two design efforts are underway: eRHIC and CBETA. eRHIC is a proposed upgrade to the existing Relativistic Heavy Ion Collider (RHIC), which would allow collisions of up to 18 GeV polarized electrons with protons or heavy ions. CBETA is a 150 MeV electron accelerator, aiming to demonstrate essential technology necessary for eRHIC.

CBETA employs FFAG arcs and is designated to use permanent magnet material for the required quadrupole magnets; this is also one option for eRHIC. One proposed design is a Halbach magnet; this paper investigates the feasibility of this approach.

INTRODUCTION

eRHIC is a proposed upgrade to the existing RHIC facility at Brookhaven National Laboratory, which would allow to collide spin-polarized protons with up to 18 GeV electrons. Part of eRHIC could be FFAG (fixed field alternating gradient) arcs, which could lower the cost of the facility.

Recently the CBETA collaboration was established between Cornell University and Brookhaven National Lab to prototype essential technology to reduce the risk for eRHIC. CBETA will accelerate electrons up to 150 MeV using a recirculating LINAC and FFAG arcs. CBETA is described in more detail in [1].

Due to the fixed magnetic field in the arcs it is attractive to utilize permanent magnets instead of a more conventional approach. One discussed approach is Halbach magnets, where the field is generated by permanent magnet blocks with varying magnetization angle.

Essential for FFAG magnets is a small magnet to magnet variation (of the order of 10−4), while the absolute field quality can be significantly worse. The magnet to magnet tolerance is dominated by the geometric tolerances as well as the tolerance of the permanent magnet blocks.

The permanent magnets are expected to be the components in a Halbach magnet design with the largest tolerances. This applies to the magnetic properties as well as the geometric tolerances. Several vendors confirmed a tolerance of several degrees for the magnetic axis of the permanent magnets (usually ±3%) and several percent of $B_r$. The expected geometric tolerance of the permanent magnets is 50 μm. Tighter tolerances can be obtained, but will drive the cost.

The aim of this paper is to estimate the effect of the permanent magnet tolerances on the gradient quality.

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† hwitte@bnl.gov

SIMULATION DETAILS

To investigate this we study the Halbach geometry shown in Fig. 1. The Halbach magnet consists of twelve sections, each magnetized in one particular direction suitable to generate a quadrupole field. Each section is a polygon, described by four points. A 1 mm gap is assumed between the individual blocks for structural support. We assume NdFeB permanent magnets with a $B_r$ of 1T. The inner clear radius is 30 mm and the outer radius is 54 mm. The targeted good field region is ±15 mm. Since the beam is small in vertical direction, the important measure for the quality of the magnet is the gradient quality on the centre plane.

![Figure 1: Geometry of the Halbach magnet used in the simulations.](image1)

The 2D commercial finite element code COMSOL1 is employed to study this. The code solves for the magnetic vector potential using vector elements. The field gradient is evaluated directly in COMSOL; since the vector elements are evaluated directly in COMSOL; since the vector elements

![Figure 2: Gradient quality (ΔG/G₀) assuming a perfect geometry and magnetic properties.](image2)

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1 COMSOL AB, Tegnergatan 23, SE-111 40 Stockholm, Sweden

07 Accelerator Technology
T09 Room-temperature Magnets
used in the simulation only evaluate the first derivative, the magnetic field is mapped to a separate variable using an additional equation. The new variable uses Lagrange elements, which compute first and second derivatives. The approach is described in [2].

The expected gradient quality without considering any tolerances is shown in Fig. 2. As shown in the figure, the gradient quality is within $1 \times 10^{-4}$; the expected gradient is 27.5 T/m.

For the following studies we introduce tolerances to the magnetic and geometric properties; the effect on the the gradient quality $\Delta G/G_0$ on the centre plane of the magnet is noted. We consider four tolerance cases, which are summarized in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical tol.</td>
<td>±25</td>
<td>±50</td>
<td>±50</td>
<td>±50 μm</td>
</tr>
<tr>
<td>Mag. Angle</td>
<td>±0.5</td>
<td>±1</td>
<td>±1</td>
<td>±1 º</td>
</tr>
<tr>
<td>$B_t$</td>
<td>±0.5</td>
<td>±1</td>
<td>±1</td>
<td>±1 %</td>
</tr>
<tr>
<td>Block align.</td>
<td>0</td>
<td>0</td>
<td>±50</td>
<td>±100 μm</td>
</tr>
<tr>
<td>Block angle</td>
<td>0</td>
<td>0</td>
<td>±0.2</td>
<td>±0.5 º</td>
</tr>
</tbody>
</table>

Cases 1 and 2 focus on variations of the permanent magnet tolerances. Cases 3 and 4 incorporate additional mechanical tolerances for the assembly.

All parameters are varied randomly in the finite element program using the inbuilt COMSOL randomizing function. For the mechanical tolerance of the permanent magnets each coordinate of each polygon is allowed to vary within the specified limit. For the alignment tolerances each block is then shifted in x/y-direction within the stated tolerances and rotated about the centre of each block.

For the analysis only the gradient quality is considered, not an absolute change of gradient. For each case 1000-3000 random variations are run; for the histogram plots of the following sections only the first 1000 variations are considered, which was found to be sufficient data for statistics.

The effect of reducing the good field region (to 80% and 60%) was also studied; in practise for this application this would be equivalent to a larger magnet with larger inner and outer radii.

**SIMULATION RESULTS**

The results of the simulation are shown in Fig. 3. The figure shows the expected yield in 10 bins for the different cases. The figure shows that gradient errors of up to 4% are predicted (for Case 4). The figure also shows that including assembly tolerances has not a large effect on the expected gradient quality variation; the magnetic and geometric tolerances of the blocks alone seem to be the determining factor in the results. Only a small percentage of the magnets are predicted in the low $10^{-3}$ gradient quality range.

**EFFECT OF REDUCING THE APERTURE**

Figure 4 shows the effect of reducing the good field region of the magnet to 80% and 60%. For this comparison we consider case 3 from table 1. As shown in the figure, depending on the reduction of the good field region the results improve significantly. This is true for both the maximum expected error as well as the yield for magnets with better gradient quality. For example, the counts in the category up to $4 \times 10^{-3}$ is more than twice in comparison to the original result if the good field region is reduced to 60% (18 mm).

**IMPROVING THE FIELD QUALITY – WIRE CORRECTION**

It has been shown that the magnetic field in the aperture of Halbach magnets can be improved by placing wires of soft-magnetic material into the aperture [3]. We demonstrate the effect on a single case of the Halbach magnet under investigation with a gradient quality of 2.35% initially.

We consider 36 potential locations of wires (in 10º increments) arranged in a circle at a radius of 27 mm; each wire
Figure 5: Gradient quality of one particular Halbach geometry; soft-iron wires are placed at the locations indicated in blue. The gradient quality is improved from 2% to 0.25%.

has a diameter of 2 mm. A Nelder-Mead algorithm is used to determine which position should be populated. Figure 5 shows the effect of the correction scheme, which suggests that two locations should be populated. The gradient quality in this example improves by almost an order of magnitude to 0.25%.

**TEMPERATURE VARIATION**

It is well known that NdFeB has a negative temperature coefficient ($\alpha = -1.1 \times 10^{-3} \degree C$). Temperature changes will therefore cause an absolute change in gradient (of about 0.1% per $\degree C$). In addition to this the gradient quality can also be affected if the temperature change is not uniform.

Figure 6 shows a simulation to study this; in the study a linear temperature gradient of $1 \degree C$ is applied across the magnet; the angle of the temperature gradient is varied and the effect on the gradient quality is monitored.

Figure 7 shows the result of the simulation; the figure shows that depending on the angle a $1 \times 10^{-3}$ error can be incurred.

**CONCLUSION**

Simulations suggest that geometric and magnetic tolerances of common NdFeB material can lead to severe field errors in Halbach magnets. Depending on the permanent magnet quality, errors of several percent can be expected.

Reducing the good field region (a larger ratio of inner magnet radius to good field region) can significantly improve the field quality.

The suggested wire scheme can correct at least part of these errors; better results should be obtainable by using more wires with the same or different cross-sections.

Temperature fluctuations and gradients are best to be avoided; while absolute temperature changes in principle can be compensated with corrector magnets, temperature gradients will lead to gradient errors. For the geometry under consideration errors up to $1 \times 10^{-3}$ were predicted.

**REFERENCES**

